Markov Chains
Toolbox

- Search: uninformed/heuristic/local
- Constraint satisfaction problems
- Probability
- Bayes nets
  - Naive Bayes classifiers
Reasoning over time

• In a Bayes net, each random variable (node) takes on one specific value.
  – Good for modeling static situations.

• What if we need to model a situation that is changing over time?
Example: Comcast

- In 2004 and 2007, Comcast had the worst customer satisfaction rating of any company or gov't agency, including the IRS.
- I have cable internet service from Comcast, and sometimes my router goes down. If the router is online, it will be online the next day with prob=0.8. If it's offline, it will be offline the next day with prob=0.4.
- How do we model the probability that my router will be online/offline tomorrow? In 2 days?
Example: Waiting in line

• You go to the Apple Store to buy the latest iPhone. Every minute, the first person in line is served with prob=0.5.
• Every minute, a new person joins the line with probability
  1 if the line length=0
  2/3 if the line length=1
  1/3 if the line length=2
  0 if the line length=3
• How do we model what the line will look like in 1 minute? In 5 minutes?
Markov Chains

- A Markov chain is a type of Bayes net with a potentially infinite number of variables (nodes).
- Each variable describes the state of the system at a given point in time.
Markov Chains

- Markov property:
  \[ P(X_t \mid X_{t-1}, X_{t-2}, X_{t-3}, \ldots) = P(X_t \mid X_{t-1}) \]

- Probabilities for each variable are identical:
  \[ P(X_t \mid X_{t-1}) = P(X_1 \mid X_0) \]
Markov Chains

• Since these are just Bayes nets, we can use standard Bayes net ideas.
  – Shortcut notation: $X_{i:j}$ will refer to all variables $X_i$ through $X_j$, inclusive.

• Common questions:
  – What is the probability of a specific event happening in the future?
  – What is the probability of a specific sequence of events happening in the future?
An alternate formulation

• We have a set of states, S.
• The Markov chain is always in exactly one state at any given time $t$.
• The chain transitions to a new state at each time $t + 1$ based only on the current state at time $t$.

$$p_{ij} = P(X_{t+1} = j \mid X_t = i)$$

• Chain must specify $p_{ij}$ for all $i$ and $j$, and starting probabilities for $P(X_0 = j)$ for all $j$. 
What is the probability my router is offline for 3 days in a row?

- \( P(X_0=\text{off}, X_1=\text{off}, X_2=\text{off}) \)?
- \( P(X_0=\text{off}) \cdot P(X_1=\text{off}|X_0=\text{off}) \cdot P(X_2=\text{off}|X_1=\text{off}) \)
- \( P(X_0=\text{off}) \cdot p_{\text{off,off}} \cdot p_{\text{off,off}} \)

\[
P(x_0:t) = P(x_0) \prod_{i=1}^{t} P(x_i | x_{i-1})
\]
More Comcast

• What is the probability my router will be offline 2 days in the future?
  – $P(X_0=\text{off})$
  – $P(X_1=\text{off}) = P(X_1=\text{off}, X_0=\text{on}) + P(X_1=\text{off}, X_0=\text{off})$
  – $P(X_1=\text{off}) = P(X_1=\text{off}|X_0=\text{on})P(X_0=\text{on})$
    + $P(X_1=\text{off}|X_0=\text{off})P(X_0=\text{off})$

$$P(x_t) = \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$