Markov Chains

Toolbox

- Search: uninformed/heuristic/local
- Constraint satisfaction problems
- Probability
- Bayes nets
  - Naive Bayes classifiers

Reasoning over time

- In a Bayes net, each random variable (node) takes on one specific value.
  - Good for modeling static situations.

- What if we need to model a situation that is changing over time?

Example: Comcast

- In 2004 and 2007, Comcast had the worst customer satisfaction rating of any company or gov't agency, including the IRS.
- I have cable internet service from Comcast, and sometimes my router goes down. If the router is online, it will be online the next day with prob=0.8. If it's offline, it will be offline the next day with prob=0.4.
- How do we model the probability that my router will be online/offline tomorrow? In 2 days?

Example: Waiting in line

- You go to the Apple Store to buy the latest iPhone. Every minute, the first person in line is served with prob=0.5.
- Every minute, a new person joins the line with probability

  1 if the line length=0

  2/3 if the line length=1

  1/3 if the line length=2

  0 if the line length=3
- How do we model what the line will look like in 1 minute? In 5 minutes?

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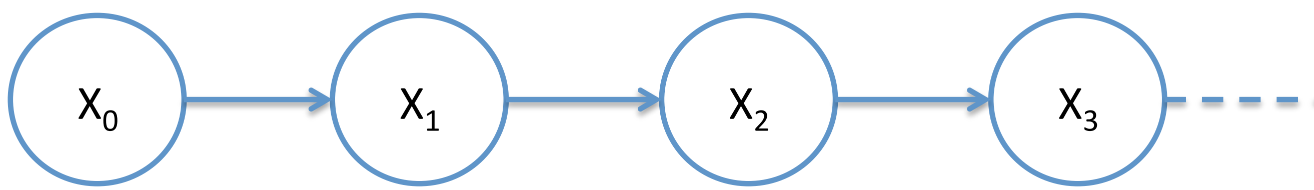
- A Markov chain is a type of Bayes net with a potentially infinite number of variables (nodes).
- Each variable is describes the state of the system at a given point in time.



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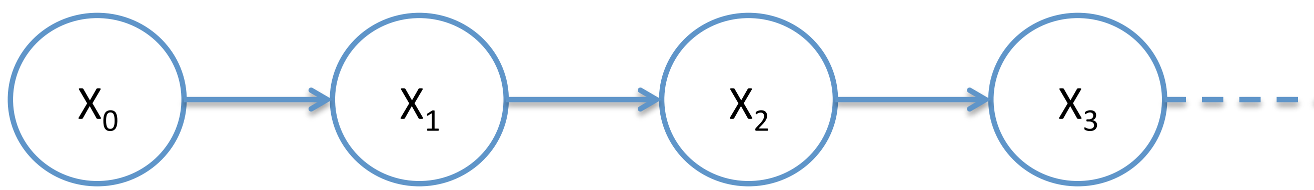
- Markov property:
  $P(X_t \mid X_{t-1}, X_{t-2}, X_{t-3}, \ldots) = P(X_t \mid X_{t-1})$
- Probabilities for each variable are identical:
  $P(X_t \mid X_{t-1}) = P(X_1 \mid X_0)$



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- Since these are just Bayes nets, we can use standard Bayes net ideas.
  - Shortcut notation: $X_{i:j}$ will refer to all variables X_i through X_j, inclusive.
- Common questions:
  - What is the probability of a specific event happening in the future?
  - What is the probability of a specific sequence of events happening in the future?

An alternate formulation

- We have a set of states, S.
- The Markov chain is always in *exactly one* state at any given time t.
- The chain transitions to a new state at each time t+1 based only on the current state at time t.

$$p_{ij} = P(X_{t+1} = j \mid X_t = i)$$

- Chain must specify p_{ij} for all i and j, and starting probabilities for $P(X_0 = j)$ for all j.

Comcast

- What is the probability my router is offline for 3 days in a row?
  - P(X0=off, X1=off, X2=off)?
  - P(X0=off)*P(X1=off|X0=off)*P(X2=off|X1=off)
  - P(X0=off)*$p_{off,off}$ * $p_{off,off}$

$$P(x_{0:t}) = P(x_0) \prod_{i=1}^{t} P(x_i \mid x_{i-1})$$

More Comcast

- What is the probability my router will be offline 2 days in the future?
  - P(X0=off)
  - P(X1=off) = P(X1=off, X0=on) + P(X1=off, X0=off)
  - P(X1=off) = P(X1=off|X0=on)P(X0=on)
    + P(X1=off|X0=off)P(X0=off)

$$P(x_t) = \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$