- **Warmup**: On paper, write a C++ function that takes a single int argument (n) and returns the product of all the integers between 1 and n.
  - Use a **for** loop.

- (This is actually a useful function in science and mathematics, called the factorial function.)
- Compare with your neighbor to see if you did it the same way.

Recursion

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  - Use a **for** loop.

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```
long long fact(int n) {
  long long answer = 1;
  for (int x = 1; x <= n; x++) {
    answer *= x;
  }
  return answer;
}
```

- Let's look at this problem a different way:
- fact(1) = 1
- fact(2) = 1 * 2
- fact(3) = 1 * 2 * 3
- fact(4) = 1 * 2 * 3 * 4
- fact(5) = 1 * 2 * 3 * 4 * 5
- Notice that each product involves computing the entire product on the row above.

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- Let's reformulate the definition of a factorial to take advantage of this.

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- Notice how for n >= 2, each factorial is defined in terms of a smaller factorial.
- So if n >= 2, what is fact(n)?
  - fact(n) = fact(n-1) * n

Recursion

- A ***recursive function*** is a function that calls itself.
- Recursive functions are used to solve problems where the ***solution to the problem may involve solving a smaller version of the same problem***.

- A recursive function has two parts:
- **Base case**: How to solve the smallest version(s) of the problem that we care about.
- **Recursive case**: How to reduce a bigger version of the problem to a smaller version.
  - In order to work, the recursive case (when applied over and over) must eventually reduce every size of the problem down to the base case.
- What are these for factorial?
- Let's write this in C++.

How does this work in C++?

- Recursion works (in all modern programming languages) because:
  - All variables are local.
  - We get new memory for local variables every time a function is called.
- Lets look at a memory diagram when we call factrec(3).

Why is this useful?

- Any loop (for/while) can be replaced with a recursive function that does the same thing.
  - Some languages don't include loops!
- Because we started with Python and C++, we naturally see things in terms of loops.
- Some problems have a "naturally" recursive solution that is hard to solve with a loop.
- Other problems have solutions that work equally well ***recursively*** or with loops (***iteratively***).

Demo

How to "get" recursion

- Forget all loops.
- To find the base case:
  - "What is the smallest version of this problem I would ever care about solving?"
- To find the recursive case:
  - "If I have a *instance* of the problem, how can I phrase how to solve the problem in terms of solving a smaller instance?"

An "instance" of a problem is a single example or occurrence of that problem.

Trust the recursion

- Base case is usually easy ("When do I stop?")
- In recursive case:
  - Break the problem into two parts (not necessarily the same size):
    - A part I can solve "now."
    - The answer from a smaller instance of the problem.
  - ***Assume the recursive call does the right thing.***
  - Figure out how to combine the two parts.

Try this

- I want to write a function that returns an uppercase version of an entire string
  - uc("hello") would return"HELLO"
- All C++ gives me is a function that returns the uppercase of a single character (toupper).
- To solve this recursively, find the recursive case and the base case.