

# Recursion II

# Factorial (iterative)

```
long long fact(int n)
{
    long long answer = 1;
    for (int x = 1; x <= n; x++)
        answer *= x;
    return answer;
}
```

# Factorial

- $\text{fact}(1) = 1$
- $\text{fact}(2) = 1 * 2$
- $\text{fact}(3) = 1 * 2 * 3$
- $\text{fact}(4) = 1 * 2 * 3 * 4$
- $\text{fact}(5) = 1 * 2 * 3 * 4 * 5$

- Let's look at this problem a different way:
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- General formula:

- **$\text{fact}(n) = 1$**  [ for  $n = 1$  ]

- **$\text{fact}(n) = \text{fact}(n-1) * n$**  [ for  $n > 1$  ]

# Factorial (recursive)

```
long long fact(n)
{
    if (n == 1)
        return 1;
    else
        return fact(n-1) * n;
}
```

# Uppercase (iterative)

```
string uc(string s)
{
    string answer = "";
    for (int x = 0; x < s.size(); x++)
        answer += toupper(s);
    return answer;
}
```

```
long long fact(int n)
{
    long long answer = 1;
    for (int x = 1; x <= n; x++)
        answer *= x;
    return answer;
}
```

# Uppercase

`uc("d") = "D"`

`uc("cd") = "CD"`

`uc("bcd") = "BCD"`

`uc("abcd") = "ABCD"`



# Uppercase

(toup stands for toupper)

$uc("d") = \text{toup}('d')$

$uc("cd") = \text{toup}('c') + \text{toup}('d')$

$uc("bcd") = \text{toup}('b') + \text{toup}('c') + \text{toup}('d')$

$uc("abcd") = \text{toup}('a') + \text{toup}('b') + \text{toup}('c') + \text{toup}('d')$

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$uc("abcd") = \text{toup}('a') + uc("bcd")$

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# Uppercase

(toup stands for toupper)

$uc("d") = toup('d')$

$uc("cd") = toup('c') + uc("d")$

$uc("bcd") = toup('b') + uc("cd")$

$uc("abcd") = toup('a') + uc("bcd")$

## **General rule:**

$uc(s) = toup(s)$  [ if s is one letter long ]

$uc(s) = toup(s[0]) + uc(\text{rest of } s)$  [ otherwise ]



Let's look at the C++ solution

# What does this do?

```
void weird(int n)
{
    if (n == 0)
        return;
    else
    {
        cout << n << endl;
        weird(n - 1);
    }
}
```

# Tower of Hanoi