Discrete Structures, Fall 2013, Homework 10

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = x^3 - 1$.

   (a) Is f 1-1? Prove or give a counterexample.

   (b) Is f onto? Prove or give a counterexample.

2. Define $g : (\mathbb{Z}^+ \times \mathbb{Z}^+) \to \mathbb{Z}^{\geq 0}$ by the rule $g(x, d) =$ the remainder when x is divided by d.

   (a) Is g 1-1? Prove or give a counterexample.

   (b) Is g onto? Prove or give a counterexample.

3. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions. Define $(f + g) : \mathbb{R} \to \mathbb{R}$ by the rule $(f + g)(x) = f(x) + g(x)$.

   (a) If f and g are both 1-1, is it true that $f + g$ is 1-1? Prove or give a counter-example.

   For a counter-example, you can just define what you want f and g to be and state (without proof) that f and g individually are 1-1 but $f + g$ is not 1-1.

   (b) If f and g are both onto, is it true that $f + g$ is onto? Prove or give a counter-example.

   For a counter-example, you can just define what you want f and g to be and state (without proof) that f and g individually are onto but $f + g$ is not onto.

4. Let $A = \{1, 2, 3, 4\}$. Define a function $f : A \to A$ using an arrow diagram such that f is 1-1 and onto, f **is not** the identity function, but $f \circ f$ **is** the identity function.

5. Let X, Y, and Z be sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, is is true that f is 1-1? Prove or give a counter-example.

6. Let X, Y, and Z be sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, is is true that g is 1-1? Prove or give a counter-example.

7. Prove that the set $S = \{1, 4, 9, 16, 26\}$ has the same cardinality as the set $\mathbb{Z}^+$.

   Hint: To prove this, you must find a 1-1 correspondence (a bijection) between $\mathbb{Z}^+$ and S. Define such a function and prove your function is 1-1 and onto.

Suggestion/hint/idea for 5 and 6: Make up some arrow diagrams first to try to work out if 5 and 6 are true or if you should find a counter-example. Note that an arrow diagram suffices for a counter-example (since it defines a function), but in general, an arrow diagram will not suffice for universal proof of a function property.

If you choose to supply a counter-example for 5 and/or 6, you don't have to show that your functions that make up the counter-example are 1-1 or not 1-1; I'll take your word for it.