Discrete Structures, Fall 2013, Homework 11

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Be aware that partial credit can only be given for a problem if your work is shown.

For these problems, you do not need to reduce factorials, powers, or "choose" notation. In fact, it's easier to grade if you don't reduce.

1. Suppose that each child born in the world is equally likely to be a boy or a girl. Consider a family with exactly three children. Let BBG indicate that the first two children born are boys and the third child is a girl, let GBG indicate that the first and third children born are girls and the second is a boy, and so forth.

   (a) List the eight elements in the sample space whose outcomes are all possible genders of the three children.

   (b) Write each of these events as a set and find its probability:
   Event X = The event that exactly one child is a girl.
   Event Y = The event that at least two children are girls.
   Event Z = The event that no child is a girl.

2. "Musical chairs" is a children's game often played at parties. If there are n children at the party, then the game starts with $n - 1$ chairs placed in a row. The children walk around the line of chairs while some music is being played. When the music stops, all the children must immediately sit down in any one of the chairs – only one person to a chair. Obviously, there will be one person who doesn't get a chair. That person is now out of the game, one chair is removed from the row, and the music and walking begin again. This continues, with one person being eliminated and one chair removed each round, until there is only one person left, who is declared the winner.

   (a) If there are 5 children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (There will be one person who doesn't get a chair.)

   (b) If there are n children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (There will be one person who doesn't get a chair.)

   (c) If there are 6 children at the party, one of whom is named Max, in how many different orders can they be eliminated during the game if you know that Max will be eliminated first? (All the children, including the winner, must be in the order.)

   (d) If there are n children at the party, one of whom is named Mindy, in how many different orders can they be eliminated during the game if you know that Mindy will be the winner? (All the children, including the winner, must be in the order.)

(e) Suppose there are four children playing: John, Kate, Lisa, and Mike. You know that John is a better at this game than Mike, and Lisa is better than Kate. Suppose you also know that in playing musical chairs, if player X is better than player Y, then Y will be eliminated before X. Using this knowledge, draw a possibility tree showing the possible orders in which the children can be eliminated (so the winners will be at the leaves of the tree).

How many different orders are there?

3. Suppose a group of six students attend a concert together.

(a) How many different ways can they be seated in a row?

(b) Suppose one of the six has to leave the concert early to finish a CS172 homework assignment. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

(c) Suppose the six students consist of three boyfriend-girlfriend couples and each couple wants to sit together so that the boy is on the right. How many ways can the six be seated?

(d) Suppose the six students consist of three math majors and three CS majors. Each group of majors wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

4. A group of eight CS172 students are all attending the movies together.

(a) How many ways can the eight people sit in a row of eight seats if two of the people are a couple and must sit side-by-side?

(b) How many ways can the eight people sit in a row of eight seats if two of the people are an ex-couple and refuse to sit side-by-side?

5. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50, how many possible combinations are there?

6. Let S be the set $\{0, 1, 2, 3, \ldots, 2n\}$ where n is some (arbitrary) positive integer.

(a) If I choose $n+1$ integers from S, must at least one of them be odd? Why or why not?

(b) If I choose $n+1$ integers from S, must at least one of them be even? Why or why not?

7. A friend in CS142 tells you that they wrote 500 lines of C++ code in 17 days. What is the largest number of lines of code they must have written in a single day? Explain.

8. Rhodes is going to send a group of computer science majors to a local high school to talk to the high schoolers about how cool CS is.

   (a) There are 20 CS majors. How many ways can a group of 5 be picked to visit the school?

   (b) The 20 CS majors consist of 12 first/second-year students and 8 third/fourth-year students. The group of 5 to visit the school should consist of at least one first/second-year student and at least one third/fourth-year student. How many ways can the group be picked?

   Hint: Use the difference rule or the addition rule.

   (c) A group of 5 is picked at random (not following the guidelines from part (b)). What is the probability that the group chosen consists of all first/second-years or all third/fourth years?