Discrete Structures, Fall 2013, Homework 5

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For each statement below, state whether it is true or false. Then prove the statement if it is true, or its negation if it is false.

Remember, an example may only be used to prove that an existential statement is true or a universal statement is false. Any example or counter-example must include specific values for the variables and enough algebra and justification to illustrate that the example proves what you are claiming it proves.

You do not need to translate each statement into symbols first, though it is often useful to do so.

1. The product of any two odd integers is odd.
2. For all integers m and n, if $m - n$ is even, then $m^3 - n^3$ is even.

   Hint: Factor $m^3 - n^3$.
3. For all integers n, if n is prime, then $(-1)^n = -1$.
4. For any rational numbers r and s, if $r < s$, then there exists a rational number t such that $r < t < s$.

   Hint: For this proof, it's helpful to translate the original statement into symbols. Notice that it will start out like a universal proof, but there's an existential inside (the variable t). Since you're just trying to show t exists, you can define t however you like.

   Hint 2: Consider $(r + s)/2$.

   It's worth noting that this is an example of a statement that is true for the rationals but not the integers.
5. For any integers a, b, and c, if $a + b = c$ and $a \mid b$, then $a \mid c$.
6. $\forall a, b, c \in \mathbb{Z}\ [(a \mid c) \wedge (b \mid c)] \rightarrow [(a \mid b) \vee (b \mid a)]$.
7. $\forall a, b, c \in \mathbb{Z}\ [(a \mid b) \wedge (a \mid c)] \rightarrow [(c \mid b) \vee (b \mid c)]$.
8. If x is an odd integer, then $x^2 - 1$ is divisible by 4.