Discrete Structures, Fall 2013, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Write out the first four terms for each of the following sequences. List the name of the variable, the subscript, and the number itself. For example, for "$\forall n \in \mathbb{Z}^{\geq 1}\ d_n = 2n$" you would write "$d_1 = 2,\ d_2 = 4,\ d_3 = 6,\ d_4 = 8.$"

   (a) $\forall i \in \mathbb{Z}^{\geq 2}\ a_i = i(i-1)$

   (b) $\forall j \in \mathbb{Z}^{\geq 0}\ s_j = \dfrac{j}{j!}$

   (c) $\forall k \in \mathbb{Z}^{+}\ z_k = (1-k)(k-1)$

2. Reduce each of the following expressions to a single numeric value.

   (a) $\displaystyle\sum_{j=1}^{5} \frac{(-1)^j}{j}$

   (b) $\displaystyle\prod_{k=0}^{10} \frac{10-k}{2^k}$

   (c) $\displaystyle\prod_{i=1}^{3} \left(\sum_{j=i}^{3} i \cdot j \right)$

3. Change the following sums into sum (sigma) notation.

   (a) $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!}$

   (b) $\dfrac{n}{1} + \dfrac{n-1}{2} + \dfrac{n-2}{3} + \cdots + \dfrac{1}{n}$

   (c) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$

4. Prove $\forall n \in \mathbb{Z}^{+}\ \displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Do this by induction. Explicitly define $P(n)$, label your base case, the inductive case, where you *define* the inductive hypothesis, and where you *use* the inductive hypothesis.