Discrete Structures, Fall 2013, Homework 9

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For each of the following, give a proof of the statement if it is true, or a counterexample if the statement is false. Remember, counterexamples must include specific values and enough work shown to demonstrate that they are actual counterexamples.
IN OTHER WORDS, NOT ALL OF THESE ARE TRUE!

1. For all sets A, B, and C, if $A \cup C = B \cup C$, then $A = B$.

2. For all sets A, B, C, and D, if $C \subseteq A$ and $(B - A)^c \subseteq D^c$, then $C \cap B \subseteq A - D$.

   Hint: Use the element method for this.

3. For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \times B \subseteq B \times C$.

   Hint: Because we're proving a Cartesian product is a subset of another Cartesian product, after you begin with "Assume $A \subseteq B$ and $B \subseteq C$," you next write "Let (x, y) be an arbitrary element in U. Assume $(x, y) \in A \times B$." Normally this line would be "Let x be an arbitrary element in $A \times B$," but because we're dealing with Cartesian products, we need that second variable (y).

4. For all sets A and B, if $A \cap B = \emptyset$ then $A \times B = \emptyset$.

5. For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.

6. For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

   Hint: This one is slightly tricky. There will be a point where you have "$X \subseteq$ something $\vee$ $X \subseteq$ something else" and you will need to divide your proof into two cases, one for each side of the or, then use dilemma/division into cases at the end to put them back together.

7. For all sets A and B, $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

   Hint: Use an algebraic proof for this.