

Discrete Structures, Fall 2014, Exam 1 Practice Problems

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Translate each of the following English sentences into formal language – that is, using the symbols \forall, \exists, \in , etc. Use the following predicates: $E(s)$ means “ s is an economics major,” $C(s)$ means “ s is a computer science major,” and $M(s)$ means “ s is a math major.” Use the domain $D =$ the set of all students at Rhodes College.
 - (a) There is an economics major who is also a math major.
 - (b) Every computer science major is also an economics major.
 - (c) No computer science majors also major in economics.
 - (d) Some people majoring in CS are also majoring in math.
 - (e) Some computer science majors are economics majors as well, but some are not.

2. Translate each of the following English sentences into formal language – that is, using the symbols \forall, \exists, \in , etc. You may define any predicates you wish, but you must list out what they mean along with your answers. (*You can define a predicate by saying something like: $J(z)$ means “ z likes to play jazz piano.”*)

Then write the negation of each statement in English, then translate the negation into formal language. (*So for each problem below, you should be writing three separate parts.*)

In your formal language statements, you may only use the domain $P =$ the set of all people.

- (a) All people are tall.
 - (b) All basketball players are tall.
 - (c) Some people speak both English and Spanish. *Use two different predicates in this problem.*
3. Let $P(x, y)$ mean “person x plays instrument y ,” let S be the set of all people, and let I be the set of all musical instruments.

Translate each of the following into English statements. Make your sentences as natural-sounding as possible, while still being precise in meaning.

- (a) $\forall x \in S \forall y \in I P(x, y)$
- (b) $\exists x \in S \exists y \in I P(x, y)$
- (c) $\forall x \in S \exists y \in I P(x, y)$
- (d) $\exists x \in S \forall y \in I P(x, y)$

- (e) $\forall y \in I \exists x \in S P(x, y)$
- (f) $\exists y \in I \forall x \in S P(x, y)$

4. In this problem, you are given a number of statements in English about people and musical instruments. You are also given a number of statements in predicate logic. For each of the English statements, you must decide which predicate logic statements are true for the English statement in question.

Here are the predicate logic statements you can pick from:

Let L be the set of people “Kate, Lisa, John,” let M be the set of musical instruments “piano, trumpet, accordion”, and let the predicate $P(x, y)$ mean “person x plays instrument y .”

- 1. $\forall x \in L \exists y \in M P(x, y)$
- 2. $\exists x \in L \forall y \in M P(x, y)$
- 3. $\forall y \in M \exists x \in L P(x, y)$
- 4. $\exists y \in M \forall x \in L P(x, y)$

You may assume that in each situation, each person plays only the instruments listed for him or her, and no others. In other words, if its not listed, they don't play it!

Here are the English statements. For each statement, write down the corresponding numbers of all the predicate logic statements above that are true for the English statement.

- (a) John plays piano, Kate plays trumpet, and Lisa plays accordion.
- (b) John plays piano, Kate plays piano and trumpet, and Lisa plays piano and accordion.
- (c) John plays trumpet, Kate plays piano, trumpet, and accordion, and Lisa doesn't play anything.
- (d) John plays trumpet, Kate plays piano and trumpet, and Lisa plays trumpet.
- (e) John plays trumpet, Kate doesn't play anything, and Lisa plays piano and accordion.
- (f) John plays accordion, Kate plays piano and accordion, and Lisa plays piano.
- (g) John plays piano, trumpet, and accordion, Kate plays trumpet and accordion, and Lisa plays accordion.
- (h) John plays piano and trumpet, Kate plays piano and accordion, and Lisa plays piano, trumpet, and accordion.