Discrete Structures, Fall 2014, Self-graded Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For each of the following, give a proof of the statement if it is true, or a counterexample if the statement is false. Remember, counterexamples must include specific values and enough work shown to demonstrate that they are actual counterexamples. **IN OTHER WORDS, NOT ALL OF THESE ARE TRUE!**

1. Prove for all sets A, B, and C, if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

   Hint: Use an element proof (standard subset proof technique).

2. Prove for all sets A and B, $(B^c \cup (B^c - A))^c = B$.

   Hint: Use an algebraic proof.

3. Prove for all sets A, B, and C, $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

4. Prove for all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \times B \subseteq B \times C$.

   Hint: Because we're proving a Cartesian product is a subset of another Cartesian product, after you begin with "Assume $A \subseteq B$ and $B \subseteq C$," you next write "Let x and y be arbitrary elements in U. Assume $(x, y) \in A \times B$." Normally this line would be "Let x be an arbitrary element in $A \times B$," but because we're dealing with Cartesian products, we need that second variable (y).

5. Prove for all sets A and B, $(A \cup B)^c = A^c \cup B^c$.