Discrete Structures, Fall 2014, Self-graded Homework 9 and Problem Set 9

- Homework 9 consists of the problems marked with the *** stars. Problem set 9 consists of all remaining problems.
- You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.
- **Be aware that partial credit can only be given for a problem if your work is shown.**
- For any problem with a numeric solution, you may give an answer reduced to any combination of addition, subtraction, multiplication, division, exponents, factorials, and/or binomial coefficients. In other words, you do not need to reduce your answer to a single number.

1. Suppose that each child born in the world is equally likely to be a boy or a girl. Consider a family with exactly three children. Let BBG indicate that the first two children born are boys and the third child is a girl, let GBG indicate that the first and third children born are girls and the second is a boy, and so forth.

   (a) ***List the eight elements in the sample space whose outcomes are all possible genders of the three children.

   (b) ***Write each of these events as a set and find its probability:
   Event X = The event that exactly one child is a girl.
   Event Y = The event that at least two children are girls.
   Event Z = The event that no child is a girl.

2. Suppose that in a certain state, all automobile license plates have four uppercase letters followed by three digits.

   (a) ***How many different license plates are possible?

   (b) ***How many license plates could begin with A and end in 0?

   (c) How many license plates could begin with "TGIF"?

   (d) How many license plates are possible in which all the letters and digits are distinct?

   (e) How many license plates could begin with "AB" and have all letters and digits distinct?

3. Suppose a group of six students attend a concert together.

   (a) ***How many different ways can they be seated in a row?

   (b) ***Suppose one of the six has to leave the concert early to finish a CS172 homework assignment. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

   (c) Suppose the six students consist of three boyfriend-girlfriend couples and each couple wants to sit together so that the boy is on the right. How many ways can the six be seated?

(d) Suppose the six students consist of three math majors and three CS majors. Each group of majors wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

4. A group of eight CS172 students are all attending the movies together.

   (a) ***How many ways can the eight people sit in a row of eight seats if two of the people are a couple and must sit side-by-side?

   (b) How many ways can the eight people sit in a row of eight seats if two of the people are an ex-couple and refuse to sit side-by-side?

5. ***Consider a randomly-chosen seven-digit telephone number. What is the probability this telephone number has at least one repeated digit?

6. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50, how many possible combinations are there?

7. Let S be the set $\{0, 1, 2, 3, \ldots, 2n\}$ where n is some (arbitrary) positive integer.

   (a) ***If I choose $n + 1$ integers from S, must at least one of them be odd? Why or why not?

   (b) ***If I choose $n + 1$ integers from S, must at least one of them be even? Why or why not?

8. A friend in CS142 tells you that they wrote 500 lines of C++ code in 17 days. What is the largest number of lines of code they must have written in a single day? Explain.

9. Rhodes is going to send a group of computer science majors to a local high school to talk to the high schoolers about how cool CS is.

   (a) ***There are 20 CS majors. How many ways can they pick a group of 5 to send to the school?

   (b) The 20 CS majors consist of 12 first/second-year students and 8 third/fourth-year students. The group of 5 to visit the school should consist of at least 1 first/second-year student and at least one third/fourth-year student. How many ways can the group be picked?

   Hint: Use the difference rule or the addition rule.

   (c) A group of 5 is picked at random (not following the guidelines from the previous part). What is the probability it consists of all first/second-years or all third/fourth years?