Discrete Structures, Fall 2014, Problem Set 5

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For each statement below, state whether it is true or false. Then prove the statement if it is true, or its negation if it is false.

Remember, an example may only be used to prove that an existential statement is true or a universal statement is false. Any example or counter-example must include specific values for the variables and enough algebra and justification to illustrate that the example proves what you are claiming it proves.

You do not need to translate each statement into symbols first, though it is often useful to do so.

1. The product of any two consecutive integers has the form $3k$ or $3k+2$ for some integer k.

2. For any integer n, $n^2 + 5$ is not divisible by 4.

   Hint: Do this by contradiction. Use the quotient-remainder theorem, but you don't need to use $d = 4$. A smaller divisor works. Each case the Q-R theorem gives you should lead to a contradiction.

3. For any integer a, if $6 \mid (3 - a)$, then $3 \nmid (a - 2)$.

   Hint: the $\nmid$ symbol means "does not divide." It is the negation of the divides predicate: $x \nmid y \Leftrightarrow \sim(x \mid y)$.

4. For any prime number p, if $p > 3$ then p can be written as either $6q + 1$ or $6q + 5$ for some integer q.

   Hint: Use the QRT with $d = 6$. Four of the cases will lead to very quick contradictions.