Discrete Structures, Fall 2014, Problem Set 8

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Let $\mathbb{R}^{\neq 0}$ be the set of all nonzero real numbers. Define $f : \mathbb{R}^{\neq 0} \to \mathbb{R}$ by the rule $f(x) = (x+1)/x$.

   (a) Is f 1-1? Prove or give a counterexample.

   (b) Is f onto? Prove or give a counterexample.

   (c) Now define $\mathbb{R}^{\neq 1}$ to be the set of all real numbers except 1. Define $g : \mathbb{R}^{\neq 0} \to \mathbb{R}^{\neq 1}$ by the rule $g(x) = (x+1)/x$. Is g onto? Prove or give a counterexample.

2. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions. Define $(f+g) : \mathbb{R} \to \mathbb{R}$ by the rule $(f+g)(x) = f(x) + g(x)$.

   (a) If f and g are both 1-1, is it true that $f+g$ is 1-1? Prove or give a counter-example.

   For a counter-example, you can just define what you want f and g to be and state (without proof) that f and g individually are 1-1 but $f+g$ is not 1-1.

   (b) If f and g are both onto, is it true that $f+g$ is onto? Prove or give a counter-example.

   For a counter-example, you can just define what you want f and g to be and state (without proof) that f and g individually are onto but $f+g$ is not onto.

3. Let X, Y, and Z be sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, is is true that f is 1-1? Prove or give a counter-example.

   Suggestion/hint/idea: Make up some arrow diagrams first to try to work out if this is true or if you should find a counter-example. Note that an arrow diagram suffices for a counter-example (since it defines a function), but in general, an arrow diagram will not suffice for universal proof of a function property.

4. Let $A = \{1, 2, 3, 4\}$. Define a function $f : A \to A$ using an arrow diagram such that f is 1-1 and onto, f **is not** the identity function, but $f \circ f$ **is** the identity function.