

Theorem 2.1.1 — Logical Equivalences (Epp, page 35)

Given any statement variables p , q , and r , a tautology \mathbf{t} , and a contradiction \mathbf{c} , the following logical equivalences hold:

Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws:	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws:	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double negative law:	$\sim(\sim p) \equiv p$	
Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal bound laws:	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations of \mathbf{t} and \mathbf{c} :	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Table 2.3.1 — Rules of Inference (Epp, page 60)

Modus ponens	$p \rightarrow q$ p $\therefore q$	Disjunctive syllogism (Elimination)	$p \vee q$ $\sim q$ $\therefore p$	$p \vee q$ $\sim p$ $\therefore q$
Modus tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Hypothetical syllogism (Transitivity)	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	
Disjunctive addition (Generalization)	p $\therefore p \vee q$	Dilemma, or Proof by division into cases	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$	
Conjunctive simplification (Specialization)	q $\therefore p \vee q$			
Conjunctive addition (Conjunction)	$p \wedge q$ $\therefore p$ $\therefore q$	Contradiction rule	$\sim p \rightarrow \mathbf{c}$ $\therefore p$	
Closing conditional world without contradiction	p q $\therefore p \wedge q$	Closing conditional world with contradiction	$\sim p \rightarrow \mathbf{c}$ $\therefore p$	p (assumed) $q \wedge \sim q$ (derived) $\therefore \sim p$

Other Logical Equivalences and Rules of Inference

Definition of implication:	$p \rightarrow q \equiv \sim p \vee q$	$\sim(p \rightarrow q) \equiv p \wedge \sim q$	
Contrapositive rule:	$p \rightarrow q \equiv \sim q \rightarrow \sim p$		
Definition of biconditional:	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$		
Negation of quantifiers:	$\sim(\forall x P(x)) \equiv \exists x \sim P(x)$	$\sim(\exists x P(x)) \equiv \forall x \sim P(x)$	
Universal modus ponens:	$\forall x \in D, P(x) \rightarrow Q(x)$ $P(a)$ where $a \in D$ $\therefore Q(a)$	Universal modus tollens:	$\forall x \in D, P(x) \rightarrow Q(x)$ $\sim Q(a)$ where $a \in D$ $\therefore \sim P(a)$
Universal instantiation:	$\forall x \in D, P(x)$ $\therefore P(a)$ where $a \in D$	Existential instantiation*:	$\exists x \in D, P(x)$ $\therefore P(a)$ where $a \in D$
Universal generalization*:	$P(a)$ where $a \in D$ $\therefore \forall x \in D, P(x)$	Existential generalization:	$P(a)$ where $a \in D$ $\therefore \exists x \in D, P(x)$

*Remember the special circumstances required for the rules marked by the stars.