Discrete Structures, Fall 2016, Homework 11

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{a, b, c, d, e\}$. Define $f : X \to Y$ as follows: $f(1) = a$, $f(2) = b$, $f(3) = b$, $f(4) = e$, and $f(5) = d$.
   (a) Draw an arrow diagram for f.
   (b) Let $A = \{1, 2, 3\}$, $S = \{a\}$, $T = \{b, c, d\}$, and $W = \{c\}$. Find $f(A)$, $f(X)$, $f^{-1}(S)$, $f^{-1}(T)$, $f^{-1}(W)$, and $f^{-1}(Y)$. [Remember that images and pre-images/inverse images are sets!]
2. Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = x^3 - 1$.
   (a) Is f 1-1? Prove or give a counterexample.
   (b) Is f onto? Prove or give a counterexample.
3. Recall that in class we took a quiz involving the function $f : \mathbb{R}^{\neq 0} \to \mathbb{R}$ defined by the rule $f(x) = \dfrac{x+1}{x}$. The quiz asked you to show that f was 1-1, but not onto.

   Suppose we modify the co-domain of function f to obtain function $g : \mathbb{R}^{\neq 0} \to \mathbb{R}^{\neq 1}$ defined by the same rule: $g(x) = \dfrac{x+1}{x}$

   Is g onto? Prove or give a counterexample.

   Note: Remember that $\mathbb{R}^{\neq 0}$ is the set of all real numbers except 0, and $\mathbb{R}^{\neq 1}$ is the set of all real numbers except 1.
4. Let $A = \{1, 2, 3, 4\}$. Define a function $f : A \to A$ using an arrow diagram such that f is 1-1 and onto, f **is not** the identity function, but $f \circ f$ **is** the identity function.

   Hint: Draw an arrow diagram with three ovals, each one with the values of A inside. Draw your arrows, ensuring that f is the same function from the first circle to the second, and from the second to the third, and has the properties above.
5. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that f is 1-1? Prove or give a counter-example.
6. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that g is 1-1? Prove or give a counter-example.

Suggestion/hint/idea for 5 and 6: Make up some arrow diagrams first to try to work out if 5 and 6 are true or if you should find a counter-example. Note that an arrow diagram suffices for a counter-example (since it defines a function), but in general, an arrow diagram will not suffice for universal proof of a function property.

One of the situations in problems and 5 and 6 is 1-1, and the other is not. In other words, one of those problems will need a proof and one will need a counter-example.

If you choose to supply a counter-example for 5 and/or 6, you don't have to show that your functions that make up the counter-example are 1-1 or not 1-1; I'll take your word for it.