Discrete Structures, Fall 2016, Homework 12

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For any problem that requires a numerical answer (as opposed to a proof or something written in words), unless otherwise specified, you do not need to fully reduce your answer to a single number — you may leave it in a form that uses addition, subtraction, multiplication, division, permutations (i.e., $P(n, k)$ notation) and combinations (i.e., $\binom{n}{k}$ notation).

Show your work for these problems! If you make a calculation error, it is easier to give partial credit if you illustrate how you derived your answer.

1. Suppose that each child born in the world is equally likely to be a boy or a girl. Consider a family with exactly three children. Let BBG indicate that the first two children born are boys and the third child is a girl, let GBG indicate that the first and third children born are girls and the second is a boy, and so forth.

   (a) List the eight elements in the sample space whose outcomes are all possible genders of the three children.

   (b) For each event below, write out the event as a set ***and*** find the probability of the event *(reduce each probability to a fraction in lowest terms)*:

   Event X = The event that exactly one child is a girl.

   Event Y = The event that at least two children are girls.

   Event Z = The event that no child is a girl.

2. Suppose a group of six students attend a concert together and all will sit in a single row of six consecutive seats at the venue.

   (Any additional in each sub-question below only pertains to that particular sub-question, unless otherwise specified.)

   (a) How many different ways can they be seated in a row?

   (b) Suppose one of the six has to leave the concert early to finish a CS172 homework assignment. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

   (c) Suppose the six students consist of three couples. Each couple naturally wants to sit side-by-side. How many ways can the six be seated?

   (d) Suppose the six students consist of three math majors and three CS majors. Each group of majors wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

   (e) Continuing from the previous sub-question: Assume that one of the CS majors is left-handed, as is one of the math majors. After they all take their seats (as specified in the previous part), they notice that the two left-handers are side by side. What is the probability this happened by chance?

3. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50 (including 0 and 50), how many possible combinations are there?

4. Rhodes College surveyed 100 prospective employers as to which programming languages they wanted their new hires to know.

   28 employers said they wanted new hires to know Python.

   26 employers said they wanted new hires to know C++.

   14 employers said they wanted new hires to know Java.

   8 employers said they wanted new hires to know Python and C++.

   4 employers said they wanted new hires to know Python and Java.

   3 employers said they wanted new hires to know C++ and Java.

   2 employers said they wanted new hires to know all three languages.

   (a) Draw a labeled Venn Diagram corresponding to this situation (label all 8 regions).
   (b) How many employers wanted students to know at least one of the languages?
   (c) How many employers wanted students to know Java and Python, but not C++?
   (d) How many employers wanted students to know Python and C++, but not Java?

5. Rhodes is going to send a group of computer science majors to a local high school to talk to the high schoolers about how cool CS is.

   (a) There are 20 CS majors. How many ways can a group of 5 be picked to visit the school?
   (b) The 20 CS majors consist of 12 first/second-year students and 8 third/fourth-year students. The group of 5 to visit the school should consist of at least one first/second-year student and at least one third/fourth-year student. How many ways can the group be picked?

       Hint: Use the difference rule or the addition rule.
   (c) A group of 5 is picked at random (not following the guidelines from part (b)). What is the probability it consists of all first/second-years or all third/fourth years?
   (d) Two other high schools get on board and want a group of 5 CS majors to visit. So now you need to pick 3 groups of 5 students each to send to the three schools. Note that it matters which group goes to which school, but within each group, the ordering of the students doesn't matter.

       Hint: Call the schools A, B, and C. First, pick the students to visit school A. Then pick the students to visit school B. Then pick the students to visit school C.

6. Recall that in a standard deck of 52 playing cards, each card has both a *suit* and a *rank*. There are four suits, called *clubs, diamonds, hearts,* and *spades*. There are thirteen ranks. Nine of those ranks are named by the numbers 2 through 10. The remaining four ranks are called *jack, queen, king*, and *ace*.

In the poker variant of Texas Hold'em, there are multiple betting rounds. In the first round, each player is dealt two playing cards at random from a standard deck of 52 cards. For these problems, the order of the cards doesn't matter.

(a) If you are dealt two cards at random from a standard deck of 52 cards, how many possible ways can this be done? That is, how many possible two-card hands are there?

(b) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards are both face cards? (Face cards are jacks, queens, and kings. The two cards don't have to be the same face card.)

(c) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability you have a *pocket pair*, meaning the two cards are of the same rank? (e.g., two queens, two fives, two aces, etc)

(d) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your hand is *suited*, meaning the two cards are of the same suit? (e.g., two diamonds, two clubs, etc)

(e) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards match in rank and suit?

(f) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards have different ranks and different suits?

7. For this problem, assume Rhodes College has 2000 students.

   (a) Is is guaranteed that among the Rhodes students, there are two students who share the same combination of initials of their first and last names? (For instance, John Smith's initials are "JS".) Mathematically, explain why or why not.

   (b) Is it guaranteed that among the Rhodes students, there are three students who share the same initials? How about four students? Mathematically, explain why or why not for each case.

   (c) Suppose all 2000 students visit the Rat for each of the three meals of the day (in other words, all students eat every meal there). Furthermore, suppose the Rat owns 500 forks and 500 spoons, and every student uses one fork and one spoon for each meal they eat. What is the minimum number of days that the Rat has to serve meals to guarantee that two students used the exact same fork and spoon pair? Explain mathematically.

8. Suppose I pick three integers arbitrarily. Use the pigeonhole principle to explain why among those three integers, there must be a pair of integers whose difference is even. (You may state whatever facts you want about even or odd numbers without proof, as long as your statements are true.)