Discrete Structures, Fall 2016, Homework 4

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Consider the following statement:

$$\exists x \in \mathbb{Z}\ x^2 = 4$$

Which of the following are correct translations of this statement? **Write down all the letters which are correct.** Note that some of the sentences below may be factually correct, but not correct translations of the original statement.

(a) The square of each integer is 4.
(b) Some integers have squares of 4.
(c) The number x, when squared, is equal to 4, for some integer x.
(d) If x is an integer, then $x^2 = 4$. (Hint: read page 70.)
(e) Some integer has a square of 4.
(f) There is at least one integer whose square is 4.

2. Consider the following statement:

$$\forall n \in \mathbb{Z}\ \mathrm{Even}(n) \to \mathrm{Even}(n^2)$$

(You may interpret Even(x) to mean "x is an even number.")

Which of the following are correct translations of this statement? **Write down all the letters which are correct.** Note that some of the sentences below may be factually correct, but not correct translations of the original statement.

(a) All integers are even and have even squares.
(b) Given any integer that is even, the square of that integer is also even.
(c) For all integers, there are some whose square is even.
(d) Any integer that is even has an even square.
(e) If an integer is even, then its square is even.
(f) All even integers have even squares.

3. Translate each of the following English sentences into formal language – that is, using the symbols $\forall, \exists, \in$, etc. Use the following predicates: $E(s)$ means "s is an economics major," $C(s)$ means "s is a computer science major," and $M(s)$ means "s is a math major." Use the domain D = the set of all students at Rhodes College.

(a) There is an economics major who is also a math major.
(b) Every computer science major is also an economics major.
(c) No computer science majors also major in economics.
(d) Some people majoring in CS are also majoring in math.

(e) Some computer science majors are economics majors as well, but some are not. (Hint: think carefully!)

4. You are given a number of statements in English below. For each statement, do the following.

   - Translate the statement into formal language using the symbols $\forall, \exists, \in$, etc. You must use the domain P = the set of all people. Do not use any other domains. You may define any predicates you wish, but you must define what they mean along with your answers. *(You can define a predicate by writing something like: "$J(x)$: x likes to play jazz piano.")*
   - Write the negation of each statement in formal language. Do not do this by merely putting a negation sign to the left of whatever you wrote for the first part — you must push the negation as far to the right as you can, so any negation symbols only appear immediately in front of predicates.
   - Write the English translation of the negative statement. Make your English translations as natural sounding as possible, without simply writing "It is not the case that..." followed by the original statement.

   (So for each statement below, you should be writing three separate parts: the translation in symbols, the negation in symbols, and the negation in English.)

   (a) All people are tall.

   (b) All basketball players are tall.

   (c) Some people speak both English and Spanish. *Use two different predicates in this problem.*

5. Let $E(x, y)$ mean "person x enjoys class y," let S be the set of all students, and let C be the set of all computer science classes.

   Translate each of the following into English statements. Make your sentences as natural-sounding as possible, while still being precise in meaning.

   (a) $\forall x \in S\ \forall y \in C\ E(x, y)$

   (b) $\exists x \in S\ \exists y \in C\ E(x, y)$

   (c) $\forall x \in S\ \exists y \in C\ E(x, y)$

   (d) $\exists x \in S\ \forall y \in C\ E(x, y)$

   (e) $\forall y \in C\ \exists x \in S\ E(x, y)$

   (f) $\exists y \in C\ \forall x \in S\ E(x, y)$