Discrete Structures, Fall 2016, Homework 5

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Complete the following proofs using the method described in class (line numbers, rule justifications, etc).

   (a) P1: $\exists w \in D \sim M(w) \rightarrow N(w)$
   P2: $\forall x \in D \sim M(x) \vee R(x)$
   P3: $\forall y \in D \sim N(y) \rightarrow \sim R(y)$
   Prove: $\exists z \in D\ N(z)$

   (b) P1: $\forall w \in D \sim R(w) \wedge Q(w)$
   P2: $\forall x \in D\ Q(x) \rightarrow \sim (P(x) \wedge S(x))$
   P3: $\forall y \in D\ (T(y) \rightarrow R(y)) \rightarrow P(y)$
   Prove: $\forall z \in D\ S(z) \rightarrow T(z)$

   (c) P1: $\forall w \in D \sim B(w)$
   P2: $\forall x \in D\ Q(x) \rightarrow (R(x) \wedge T(x))$
   P3: $\forall y \in D\ [B(y) \rightarrow \sim Q(y)] \rightarrow [R(y) \rightarrow B(y)]$
   Prove: $\forall z \in D \sim Q(z)$

2. (a) State whether or not the argument below is valid or invalid. Recall that a valid argument has a conclusion that can be derived from the premises using only rules of inference and/or logical equivalences, whereas an invalid argument has a conclusion that cannot be derived from the premises.

   (b) Next, translate the premises into formal logic using the universal set U as your domain, with the predicates $P(x)$ = "x is a pig," $F(x)$ = "x is fat," and $S(x)$ = "x likes sleeping a lot."

   (c) Lastly, if the argument is valid, give a formal proof.

   Argument:

   P1: All pigs are fat.
   P2: Some pigs like sleeping a lot.
   Therefore, some things that are fat like sleeping a lot.

3. Suppose you are given the following premises and conclusion:

   P1: $\exists w \in D\ G(w)$
   P2: $\exists x \in D\ H(x)$
   Conclusion: $\exists z \in D\ G(z) \wedge H(z)$

   (a) Suppose you start the proof by using existential instantiation on P1 to get $G(a)$, and then you existentially instantiate P2 to get $H(a)$. You then use conjunctive addition to get $G(a) \wedge H(a)$, then existential generalization to get $\exists z \in D\ G(z) \wedge H(z)$. What was wrong in this sequence of steps?

(b) Can this proof be done at all (in other words, is it even a valid argument)? Why or why not? Give a real-world example assigning meaning to the domain D and predicates G and H to support your claim.

4. In this problem, you are given a number of statements in English about people and musical instruments. You are also given a number of statements in predicate logic. For each of the English statements, you must decide which predicate logic statements are true for the English statement in question.

   Here are the predicate logic statements you can pick from:

   Let L be the set of people "Kate, Lisa, John;" let M be the set of musical instruments "piano, trumpet, accordion;" and let the predicate $P(x, y)$ mean "person x plays instrument y."

   1. $\forall x \in L\ \exists y \in M\ P(x, y)$
   2. $\exists x \in L\ \forall y \in M\ P(x, y)$
   3. $\forall y \in M\ \exists x \in L\ P(x, y)$
   4. $\exists y \in M\ \forall x \in L\ P(x, y)$

   You may assume that in each situation, each person plays only the instruments listed for him or her, and no others. In other words, if its not listed, they don't play it!

   Here are the English statements. For each statement, write down the corresponding numbers of all the predicate logic statements above that are true for the English statement.

   (a) John plays piano, Kate plays trumpet, and Lisa plays accordian.
   (b) John plays piano, Kate plays piano and trumpet, and Lisa plays piano and accordian.
   (c) John plays trumpet, Kate plays piano, trumpet, and accordian, and Lisa doesn't play anything.
   (d) John plays trumpet, Kate plays piano and trumpet, and Lisa plays trumpet.
   (e) John plays trumpet, Kate doesn't play anything, and Lisa plays piano and accordian.
   (f) John plays accordian, Kate plays piano and accordian, and Lisa plays piano.
   (g) John plays piano, trumpet, and accordian, Kate plays trumpet and accordian, and Lisa plays accordian.
   (h) John plays piano and trumpet, Kate plays piano and accordian, and Lisa plays piano, trumpet, and accordian.