Discrete Structures, Fall 2016, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove the statements below. They are both true. Refer to section 4.4.

1. The product of any two consecutive integers is even.

   Hints: Use only one universally-quantified variable, not two. Use the quotient-remainder theorem with $d = 2$.

2. For any integers m and n, if $m = n - 4$, then $m \cdot n$ can be written as either $3k$ or $3k + 2$ for some integer k.

   Hints: Use the quotient-remainder theorem on either m or n, but not both. See if you can determine on your own what d should be. Once the QRT gives you an equation for m or n in terms of q, you should be able to derive an equation for the other variable (m or n) in terms of q as well.

Do the following problems about sequences and series. Refer to section 5.1.

3. Write out the first four terms for each of the following sequences. List the name of the variable, the subscript, and the number itself. For example, for "$\forall n \in \mathbb{Z}^{\geq 1}\ d_n = 2n$" you would write "$d_1 = 2,\ d_2 = 4,\ d_3 = 6,\ d_4 = 8$."

   (a) $\forall i \in \mathbb{Z}^{\geq 2}\ a_i = i(i-1)$

   (b) $\forall j \in \mathbb{Z}^{\geq 0}\ s_j = \dfrac{j}{j!}$

   (c) $\forall k \in \mathbb{Z}^{+}\ z_k = (1-k)(k-1)$

4. Write the following sums using sigma (Σ) notation.

   (a) $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!}$

   (b) $\dfrac{n}{1} + \dfrac{n-1}{2} + \dfrac{n-2}{3} + \cdots + \dfrac{1}{n}$

   (c) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$

5. Rewrite each of the following summations as an equivalent expression by separating off the last term. For reference, the first one has been done for you (we did this in class).

   (a) (Example:) $\displaystyle\sum_{i=0}^{k+1} i^2 = \sum_{i=0}^{k} i^2 + (k+1)^2$

   (b) $\displaystyle\sum_{p=0}^{m+1} \frac{1}{2^{p-1}}$

   (c) $\displaystyle\sum_{x=0}^{k} \frac{x+1}{x+2}$