Discrete Structures, Fall 2016, Homework 8

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove the following statements by induction . Make sure to follow the form from class: explicitly define $P(n)$, label the basis step, inductive step, where you define the inductive hypothesis, where you define what you want to prove, and where you use the inductive hypothesis.

1. Prove $\forall n \in \mathbb{Z}^+ \ \sum_{i=1}^{n}(2 \cdot i^2) = \frac{n(n+1)(2n+1)}{3}$.

2. Prove $\forall n \in \mathbb{Z}^+ \ 11^n - 6$ is divisible by 5.

   Hint: $11 = 10 + 1$.

3. Prove $\forall n \in \mathbb{Z}^+ \ \sum_{i=1}^{n}(i \cdot i!) = (n+1)! - 1$

   Hint: Recall that $n! = n(n-1)(n-2)\cdots 2 \cdot 1$, with $0!$ defined to be 1. However, an alternate formula involving recursion is the following:

   $$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

   This recursive definition will be useful because there will be a part of the inductive step where you need to rewrite $(k+2)(k+1)!$ as something different.

4. Prove $\forall n \in \mathbb{Z}^{\geq 2} \ \prod_{i=2}^{n}\left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$