Discrete Structures, Fall 2017, Homework 10

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{a, b, c, d, e\}$. Define $f : X \to Y$ as follows: $f(1) = a$, $f(2) = b$, $f(3) = b$, $f(4) = e$, and $f(5) = d$.

   (a) Draw an arrow diagram for f.

   (b) Let $A = \{1, 2, 3\}$, $S = \{a\}$, $T = \{b, c, d\}$, and $W = \{c\}$. Find $f(A)$, $f(X)$, $f^{-1}(S)$, $f^{-1}(T)$, $f^{-1}(W)$, and $f^{-1}(Y)$. [Remember that images and pre-images/inverse images are sets!]

2. Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = x^3 + 1$.

   (a) Is f 1-1? Prove or give a counterexample.

   (b) Is f onto? Prove or give a counterexample.

3. Let $A = \{1, 2, 3, 4\}$. Define a function $f : A \to A$ using an arrow diagram such that f is 1-1 and onto, f **is not** the identity function, but $f \circ f$ **is** the identity function.

   Hint: Draw an arrow diagram with three ovals, each one with the values of A inside. Draw your arrows, ensuring that f is the same function from the first circle to the second, and from the second to the third, and has the properties above.

4. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that f is 1-1? Prove or give a counter-example.

5. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that g is 1-1? Prove or give a counter-example.

Suggestion/hint/idea for the last two problems: Make up some arrow diagrams first to try to work out if 4 and 5 are true or if you should find a counter-example. Note that an arrow diagram suffices for a counter-example (since it defines a function), but in general, an arrow diagram will not suffice for universal proof of a function property.

One of the situations in the last two problems is 1-1, and the other is not. In other words, one of those problems will need a proof and one will need a counter-example.