Discrete Structures, Fall 2017, Homework 11

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Helpful hint — read sections 9.1, 9.2, and 9.3 first!

For any problem that requires a numerical answer (as opposed to a proof or something written in words), unless otherwise specified, you do not need to fully reduce your answer to a single number — you may leave it in a form that uses addition, subtraction, multiplication, division, permutations (i.e., $P(n,k)$ notation) and combinations (i.e., $\binom{n}{k}$ notation).

Show your work for these problems! If you make a calculation error, it is easier to give partial credit if you illustrate how you derived your answer.

1. There have been three computer science talks this semester (Slingshot, FedEx, and the industry panel). Suppose you chose at random whether to attend each talk (in other words, for each talk, there is a 50–50 chance that you attended). Let the letter "Y" mean you attended a talk, and "N" mean you didn't attend. So you can encode your attendance at each talk by a 3-character string of of Ys and/or Ns. For example, YNN means you attended the Slingshot talk, but not the FedEx or industry panel talks.

   (a) List the eight elements in the sample space whose outcomes are all possible ways you could have attended/not attended the talks.

   (b) Write each of these events as a set *and* find its probability *(reduce each probability to a fraction in lowest terms)*:

   Event X = The event that you attended exactly one of the talks.

   Event Y = The event that you attended at least two of the talks.

   Event Z = The event that you attended none of the talks.

2. *(For this question, reduce each answer to a single integer or to a fraction in lowest terms.)*

   (a) How many positive 3-digit numbers are multiples of 6?

   (b) What is the probability that a randomly chosen positive three-digit integer is a multiple of 6?

   (c) What is the probability that a randomly chosen positive three-digit integer is a multiple of 7?

3. Suppose that in a certain state, all automobile license plates have four uppercase letters followed by three digits.

   (a) How many different license plates are possible?

   (b) How many license plates could begin with A and end in 0?

   (c) How many license plates could begin with "TGIF"?

   (d) How many license plates are possible in which all the letters and digits are distinct?

   (e) How many license plates could begin with "AB" and have all letters and digits distinct?

4. Suppose a group of six students attend a concert together and all will sit in a single row of six consecutive seats at the venue.

   (Any additional in each sub-question below only pertains to that particular sub-question, unless otherwise specified.)

   (a) How many different ways can they be seated in a row?

   (b) Suppose one of the six has to leave the concert early to finish a CS172 homework assignment. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working student must be in the aisle seat?

   (c) Suppose the six students consist of three couples. Each couple naturally wants to sit side-by-side. How many ways can the six people be seated?

   (d) Suppose the six students consist of three math majors and three CS majors. Each group of majors wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

   (e) Continuing from the previous sub-question: Assume that one of the CS majors is left-handed, as is one of the math majors. After they all take their seats (as specified in the previous part), they notice that the two left-handers are side by side. What is the probability this happened by chance?

5. A group of eight CS172 students are all attending the movies together. Like in the previous question, they will all sit in a single row of eight consecutive seats.

   (a) How many ways can the eight people sit in a row of eight seats if two of the people are a couple and must sit side-by-side?

   (b) How many ways can the eight people sit in a row of eight seats if two of the people are a former couple and refuse to sit side-by-side?

6. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50 (including 0 and 50), how many possible combinations are there?

7. Consider a randomly-chosen seven-digit telephone number. What is the probability this telephone number has at least one repeated digit?

8. Rhodes College surveyed 100 prospective employers as to which programming languages they wanted their new hires to know.

   28 employers said they wanted new hires to know Python.

   26 employers said they wanted new hires to know C++.

   14 employers said they wanted new hires to know Java.

   8 employers said they wanted new hires to know Python and C++.

   4 employers said they wanted new hires to know Python and Java.

3 employers said they wanted new hires to know C++ and Java.

2 employers said they wanted new hires to know all three languages.

Note that when we say "28 employers wanted Python," that means that 28 total employers checked off Python on the survey. Some of those 28 may have checked off other languages as well.

(a) Draw a labeled Venn Diagram corresponding to this situation (label all 8 regions).

(b) How many employers wanted students to know at least one of the languages?

(c) How many employers wanted students to know Java and Python, but not C++?

(d) How many employers wanted students to know Python and C++, but not Java?