

Discrete Structures, Fall 2017, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove each of the following statements using “regular” or “weak” induction.

1. $\forall n \in \mathbb{Z}^{\geq 0} \sum_{i=0}^n (3i^2 - i) = n^2(n + 1)$

2. Prove $\forall n \in \mathbb{Z}^+ \prod_{i=1}^n i(i + 1) = (n + 1)(n!)^2$

Hint: Recall that $n! = n(n - 1)(n - 2) \cdots 2 \cdot 1$, with $0!$ defined to be 1. However, an alternate formula involving recursion is the following:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n - 1)! & \text{otherwise} \end{cases}$$

This recursive definition will be useful during the inductive step.

3. $\forall n \in \mathbb{Z}^{\geq 0} n(n + 1)$ is even.

Note: This is the same problem as question 1 on the last homework (prove that the product of any two consecutive integers is even). In homework 7, you did this with the QRT. On this homework, you should use induction (do not use the QRT here).

4. $\forall n \in \mathbb{Z}^{\geq 0} 5 \mid 7^n - 2^n$.