

Discrete Structures, Fall 2017, Homework 8

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove the following statements by strong induction. Make sure to follow the form from class: explicitly define $P(n)$, label the basis step(s), inductive step, where you define the inductive hypothesis, where you define what you want to prove and where you use the inductive hypothesis.

1. Suppose we define a sequence as follows:

$$a_1 = 3; a_2 = 5; \text{ and for all integers } i \geq 3, a_i = 4a_{i-1} - 3a_{i-2}.$$

Prove $\forall n \in \mathbb{Z}^{\geq 1} a_n = 3^{n-1} + 2$.

2. Suppose we define a sequence as follows:

$$b_0 = 3; b_1 = 1; b_2 = 3; \text{ and for all integers } i \geq 3, b_i = b_{i-3} + b_{i-2} + b_{i-1}.$$

Prove that every term in the sequence is odd.

3. Suppose we define a sequence as follows:

$$c_0 = 2; c_1 = 7; \text{ and for all integers } i \geq 2, c_i = 3c_{i-1} - 2c_{i-2}.$$

Prove $\forall n \in \mathbb{Z}^{\geq 0} 5 \mid (c_n - 2)$.