Discrete Structures, Fall 2017, Homework 9

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Prove the following statement using an *element proof*:

   For any four sets A, B, C, and D, if $C \subseteq (D \cup A)$ and $B \subseteq D^c$, then $C \cap B \subseteq A$.

2. Prove the following statement using an *element proof*:

   For any three sets A, B, and C, if $A \cap C \subseteq B$, then $(A - B) \cap (C - B) = \emptyset$.

3. Prove the following statement using an *element proof*:

   For any three sets A, B, and C, if $A \cup C \subseteq B$, then $C \times A \subseteq B \times B$.

4. Prove the following statement using an *algebraic proof*:

   For any two sets A and B, $(B \cup (B - A^c))^c = B^c$.

5. Answer the following questions about power sets:

   (a) Define $S = \{x, y, z\}$. What is $\mathcal{P}(S)$?

   (b) What is $\mathcal{P}(\emptyset)$?

   (c) What is $\mathcal{P}(\mathcal{P}(\emptyset))$?