Discrete Structures, Spring 2013, Homework 12

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Be aware that partial credit can only be given for a problem if your work is shown. Simplify all answers to expressions involving only factorials, addition, subtraction, multiplication, division, and exponents.

1. A group of eight CS172 students are all attending the movies together.
   (a) How many ways can the eight people sit in a row of eight seats if two of the people are a couple and must sit side-by-side?
   (b) How many ways can the eight people sit in a row of eight seats if two of the people are an ex-couple and refuse to sit side-by-side?

2. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50, how many possible combinations are there?

3. Rhodes College surveyed 100 prospective employers as to which programming languages they wanted their new hires to know.

   28 employers said they wanted new hires to know Python.

   26 employers said they wanted new hires to know C++.

   14 employers said they wanted new hires to know Java.

   8 employers said they wanted new hires to know Python and C++.

   4 employers said they wanted new hires to know Python and Java.

   3 employers said they wanted new hires to know C++ and Java.

   2 employers said they wanted new hires to know all three languages.

   (a) Draw a labeled Venn Diagram corresponding to this situation (label all 8 regions).
   (b) How many employers wanted students to know at least one of the languages?
   (c) How many employers wanted students to know Java and Python, but not C++?
   (d) How many employers wanted students to know Python and C++, but not Java?

4. Let S be the set $\{0, 1, 2, 3, \ldots, 2n\}$ where n is some (arbitrary) positive integer.
   (a) If I choose $n + 1$ integers from S, must at least one of them be odd? Why or why not?
   (b) If I choose $n + 1$ integers from S, must at least one of them be even? Why or why not?

5. A friend in CS142 tells you that they wrote 500 lines of C++ code in 17 days. What is the largest number of lines of code they must have written in a single day? Explain.

6. I pick three integers arbitrarily. Use the pigeonhole principle to explain why among those three integers, there must be a pair of integers whose difference is even. (You may state whatever facts you want about even or odd numbers without proof, as long as your statements are true.)

7. Rhodes is going to send a group of computer science majors to a local high school to talk to the high schoolers about how cool CS is.

   (a) There are 20 CS majors. How many ways can a group of 5 be picked to visit the school?

   (b) The 20 CS majors consist of 12 first/second-year students and 8 third/fourth-year students. The group of 5 to visit the school should consist of at least one first/second-year student and at least one third/fourth-year student. How many ways can the group be picked?

   Hint: Use the difference rule or the addition rule.

   (c) A group of 5 is picked at random (not following the guidelines from part (b)). What is the probability it consists of all first/second-years or all third/fourth years?

   (d) Two other high schools get on board and want a group of 5 CS majors to visit. So now you need to pick 3 groups of 5 students each to send to the three schools. Note that it matters which group goes to which school, but within each group, the ordering of the students doesn't matter.

   Hint: Call the schools A, B, and C. First, pick the students to visit school A. Then pick the students to visit school B. Then pick the students to visit school C.