Discrete Structures, Spring 2013, Homework 13

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Be aware that partial credit can only be given for a problem if your work is shown.

1. Let A and B be sets with $N(A) = m$ and $N(B) = n$. How many possible functions are there from A to B? How many possible binary relations are there from A to B?

2. Let $A = \{1, 2, 3, 4\}$.

   (a) Define a binary relation R_1 on A that is reflexive and symmetric, but not transitive.

   (b) Define a binary relation R_2 on A that is transitive and symmetric, but not reflexive.

   (c) Define a binary relation R_3 on A that is transitive and reflexive, but not symmetric.

   (Note that you don't necessarily have to define a relation with a "rule;" you can list out the ordered pairs or use an arrow diagram.)

3. For each of the following relations, decide if the relation is reflexive, symmetric, and/or transitive. Justify each decision with a proof or counterexample.

   (a) Define a relation D on set $\mathbb{Z}$ by the rule: for all integers m and n, $m \; D \; n \Leftrightarrow m \mid n$.

   (b) Define a relation R on the set $S = \{1, 2, 3, 4\}$ by the rule: for all integers $a, b \in S$, $a \; R \; b$ if and only if the binary representation of a has the same number of 1s as the binary representation of b.

   (c) Let C be the set of all propositional statements over three variables p, q, and r. (In other words, every propositional statement that involves only the variables p, q, and r is in C). Define I to be the "implies" relation on C: $(x, y) \in I$ if and only if the statement $x \rightarrow y$ is true.

   (d) Define a relation Q on the set $\mathbb{R}$ by the following rule: for all $x, y \in \mathbb{R}$, $x \; Q \; y \Leftrightarrow x - y \in \mathbb{Z}$.

   (e) Define a relation T on $\mathcal{P}(\mathbb{Z})$ by the following rule:
   for all $A, B \in \mathcal{P}(\mathbb{Z})$, $A \; T \; B \Leftrightarrow N(A) \neq N(B)$.