Discrete Structures, Spring 2013, Homework 2

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the entries in the truth table that led you to your answer, and explain why those entries support your answer.

$$\begin{array}{ll} & \sim p \vee q \\ & r \to \sim q \\ \hline \therefore & p \to \sim r \end{array}$$

2. This question allows you to practice two different ways that will verify that the following two statements are logically equivalent.

   - $p \leftrightarrow q$
   - $(q \wedge p) \vee \sim(p \vee q)$

   (a) Construct a complete truth table to show that the two statements above are logically equivalent (they are indeed logically equivalent).

   (b) Next use only the rules given in Table 2.1.1 from the book along with definitions of biconditional and implication as presented on the reference sheet to show that the two statements above are logically equivalent. Use the format of the proof shown in class – each line of your proof must be justified with one of the rules from Table 2.1.1 and you must tell which line that rule was applied to get the new line you are adding to your proof. (The only difference is that you can't use the rules of inference, only the logical equivalences.)

3. Complete the following proofs using the framework discussed in class. Each line of your proof must be justified with a rule of inference or logical equivalence and appropriate line numbers.

   (a)
$$\begin{array}{ll} \text{P1} & (p \to q) \wedge (r \to s) \\ \text{P2} & v \\ \text{P3} & (s \wedge q) \to \sim v \\ \hline \text{Prove:} & \sim p \vee \sim r \end{array}$$

   (b)
$$\begin{array}{ll} \text{P1} & p \to q \\ \text{P2} & \sim q \vee r \\ \text{P3} & s \vee (v \wedge \sim r) \\ \hline \text{Prove:} & \sim s \to \sim(p \vee \sim v) \end{array}$$

   (c)
$$\begin{array}{ll} \text{P1} & a \wedge \sim d \\ \text{P2} & b \to (e \to d) \\ \hline \text{Prove:} & (a \to b) \to \sim e \end{array}$$