Discrete Structures, Spring 2013, Homework 4

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Complete the following proofs using the method described in class (line numbers, rule justifications, etc).

   (a) P1: $\exists w \in D \sim Q(w) \vee P(w)$
   P2: $\forall x \in D\ Q(x) \vee R(x)$
   P3: $\forall y \in D\ R(y) \rightarrow P(y)$
   Prove: $\exists z \in D\ P(z)$

   (b) P1: $\forall w \in D \sim R(w) \wedge Q(w)$
   P2: $\forall x \in D\ Q(x) \rightarrow \sim(P(x) \wedge S(x))$
   P3: $\forall y \in D\ (T(y) \rightarrow R(y)) \rightarrow P(y)$
   Prove: $\forall z \in D\ S(z) \rightarrow T(z)$

   (c) P1: $\forall w \in D \sim L(w)$
   P2: $\forall x \in D\ S(x) \rightarrow (R(x) \wedge T(x))$
   P3: $\forall y \in D\ [L(y) \rightarrow \sim S(y)] \rightarrow [R(y) \rightarrow L(y)]$
   Prove: $\forall z \in D \sim S(z)$

2. State whether or not the argument below is valid or invalid. Recall that a valid argument has a conclusion that can be derived from the premises using only rules of inference and/or logical equivalences, whereas an invalid argument has a conclusion that cannot be derived from the premises.

   Next, translate the premises into formal logic using the universal set U as your domain, with the predicates $P(x)$ = "x is a pig," $F(x)$ = "x is fat," and $S(x)$ = "x likes sleeping a lot."

   Last, if the argument is valid, give a formal proof.

   Argument:

   P1: All pigs are fat.
   P2: Some pigs like sleeping a lot.
   Therefore, some things that are fat like sleeping a lot.

3. Suppose you are given the following premises and conclusion:

   P1: $\exists w \in D\ G(w)$
   P2: $\exists x \in D\ H(x)$
   Conclusion: $\exists z \in D\ G(z) \wedge H(z)$

   (a) Suppose you start the proof by using existential instantiation on P1 to get $G(a)$, and then you existentially instantiate P2 to get $H(a)$. You then use conjunctive addition to get $G(a) \wedge H(a)$, then existential generalization to get $\exists z \in D\ G(z) \wedge H(z)$. What was wrong in this sequence of steps?

   (b) Can this proof be done at all (in other words, is it even a valid argument)? Why or why not? Give a real-world example assigning meaning to the domain D and predicates G and H to support your claim.