Discrete Structures, Spring 2013, Homework 5

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove each of the following statements true or false. Remember, an example may only be used to prove that an existential statement is true or a universal statement is false. Any example or counter-example must include specific values for the variables and enough algebra and justification to illustrate that the example proves what you are claiming it proves.

1. For all integers m and n, if $m - n$ is even, then $m^3 - n^3$ is even.

   Hint: $m^3 - n^3$ factors.

2. For all integers m, if $m > 2$, then $m^2 - 4$ is composite.

   Hint: Composite$(x) \Leftrightarrow \exists r, s \in \mathbb{Z}^+ \ (x = rs) \wedge (1 < r < x) \wedge (1 < s < x)$

3. For all integers n, if n is prime, then $(-1)^n = -1$.

4. For any rational numbers r and s, if $r < s$, then there exists a rational number t such that $r < t < s$.

   Hint: consider $(r + s)/2$.

   It's worth noting that this is an example of a statement that is true for the rationals but not the integers.

5. For any integers a, b, and c, if $a + b = c$ and $a \mid b$, then $a \mid c$.

6. $\forall a, b, c \in \mathbb{Z}\ [(a \mid c) \wedge (b \mid c)] \rightarrow [(a \mid b) \vee (b \mid a)]$.

7. $\forall a, b, c \in \mathbb{Z}\ [(a \mid b) \wedge (a \mid c)] \rightarrow [(c \mid b) \vee (b \mid c)]$.

8. If x is an odd integer, then $x^2 - 1$ is divisible by 4.