Discrete Structures, Spring 2013, Homework 6

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove each of the following statements true or false. Remember, an example may only be used to prove that an existential statement is true or a universal statement is false. Any example or counter-example must include specific values for the variables and enough algebra and justification to illustrate that the example proves what you are claiming it proves.

1. For any integer n, $n^2 + 5$ is not divisible by 4.

   Hint: Do this by contradiction. Use the Q-R theorem, but you don't need to use $d = 4$. A smaller divisor works. Each case the Q-R theorem gives you should lead to a contradiction.

2. For any prime number p, if $p > 3$ then p can be written as either $6q + 1$ or $6q + 5$ for some integer q.

   Hint: You need all six cases here, but four of them lead to contradictions.

3. The square root of 5 is irrational.

4. For any integer a, if $6 \mid (3 - a)$, then $3 \nmid (a - 2)$.

   Hint: the $\nmid$ symbol means "does not divide." It is the negation of the divides predicate: $x \nmid y \Leftrightarrow \sim(x \mid y)$

5. $\log_5(2)$ is irrational.

   Hint: Do this by contradiction. Work with the equation using the laws of logarithms and exponents (see the back inside cover of the book if you need a refresher) until you get an equation with one prime number raised to a power on each side of the equals sign. Then use the unique prime factorization theorem to show that there are no numbers that can make the equation true.