Discrete Structures, Spring 2013, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Write out the first four terms for each of the following sequences.

   (a) $\forall i \in \mathbb{Z}^{\geq 2}\ a_i = i(i-1)$

   (b) $\forall j \in \mathbb{Z}^{\geq 0}\ s_j = \dfrac{j}{j!}$

   (c) $\forall k \in \mathbb{Z}^{+}\ z_k = (1-k)(k-1)$

2. Reduce each of the following expressions to a single numeric value.

   (a) $\displaystyle\sum_{j=1}^{5} \frac{(-1)^j}{j}$

   (b) $\displaystyle\prod_{k=0}^{10} \frac{10-k}{2^k}$

   (c) $\displaystyle\prod_{i=1}^{3} \left(\sum_{j=i}^{3} i \cdot j \right)$

3. Change the following sums into sum (sigma) notation.

   (a) $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!}$

   (b) $\dfrac{n}{1} + \dfrac{n-1}{2} + \dfrac{n-2}{3} + \cdots + \dfrac{1}{n}$

   (c) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$

4. Prove $\forall n \in \mathbb{Z}^{+}\ \displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Do this by induction. Explicitly define $P(n)$, label your base case, the inductive case, the inductive hypothesis, and the inductive step. Do not forget to state where you use the inductive hypothesis.