Discrete Structures, Spring 2016, Homework 10

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Let $X = \{a, b, c\}$ and $Y = \{r, s, t, u, v, w\}$. Define $f : X \to Y$ as follows: $f(a) = v$, $f(b) = v$, and $f(c) = t$.

   (a) Draw an arrow diagram for f.

   (b) Let $A = \{a, b\}$, $C = \{t\}$, $D = \{u, v\}$, and $E = \{r, s\}$. Find $f(A)$, $f(X)$, $f^{-1}(C)$, $f^{-1}(D)$, $f^{-1}(E)$, and $f^{-1}(Y)$.

2. Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = x^3 - 1$.

   (a) Is f 1-1? Prove or give a counterexample.

   (b) Is f onto? Prove or give a counterexample.

3. Let $\mathbb{R}^{\neq 0}$ be the set of all nonzero real numbers. Define $f : \mathbb{R}^{\neq 0} \to \mathbb{R}$ by the rule $f(x) = (x + 1)/x$.

   (a) Is f 1-1? Prove or give a counterexample.

   (b) Is f onto? Prove or give a counterexample.

   (c) Now define $\mathbb{R}^{\neq 1}$ to be the set of all real numbers except 1. Define $g : \mathbb{R}^{\neq 0} \to \mathbb{R}^{\neq 1}$ by the rule $g(x) = (x + 1)/x$. Is g onto? Prove or give a counterexample.

4. Let $A = \{1, 2, 3, 4\}$. Define a function $f : A \to A$ using an arrow diagram such that f is 1-1 and onto, f **is not** the identity function, but $f \circ f$ **is** the identity function.

5. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that f is 1-1? Prove or give a counter-example.

6. Let X, Y, and Z be any sets. Suppose $f : X \to Y$ and $g : Y \to Z$ are functions. If $g \circ f$ is 1-1, must it be true that g is 1-1? Prove or give a counter-example.

Suggestion/hint/idea for 5 and 6: Make up some arrow diagrams first to try to work out if 5 and 6 are true or if you should find a counter-example. Note that an arrow diagram suffices for a counter-example (since it defines a function), but in general, an arrow diagram will not suffice for universal proof of a function property.

If you choose to supply a counter-example for 5 and/or 6, you don't have to show that your functions that make up the counter-example are 1-1 or not 1-1; I'll take your word for it.