**Discrete Structures, Spring 2016, Homework 11**

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

**For any problem that requires a numerical answer (as opposed to a proof or something written in words), unless otherwise specified, you do not need to fully reduce your answer to a single number — you may leave it in a form that uses addition, subtraction, multiplication, division, permutations (i.e., $P(n,k)$ notation) and combinations (i.e., $\binom{n}{k}$ notation).**

**Show your work for these problems! If you make a calculation error, it is easier to give partial credit if you illustrate how you derived your answer.**

1. Suppose that each child born in the world is equally likely to be a boy or a girl. Consider a family with exactly three children. Let BBG indicate that the first two children born are boys and the third child is a girl, let GBG indicate that the first and third children born are girls and the second is a boy, and so forth.

   - List the eight elements in the sample space whose outcomes are all possible genders of the three children.
   - Write each of these events as a set and find its probability *(reduce each probability to a fraction in lowest terms)*:
     Event X = The event that exactly one child is a girl.
     Event Y = The event that at least two children are girls.
     Event Z = The event that no child is a girl.

2. *(For this question, reduce each answer to a single integer or to a fraction in lowest terms.)*

   (a) How many positive 3-digit numbers are multiples of 6?

   (b) What is the probability that a randomly chosen positive three-digit integer is a multiple of 6?

   (c) What is the probability that a randomly chosen positive three-digit integer is a multiple of 7?

3. "Musical chairs" is a children's game often played at parties. If there are $n$ children at the party, then the game starts with $n-1$ chairs placed in a row. The children walk around the line of chairs while some music is being played. When the music stops, all the children must immediately sit down in any one of the chairs – only one person to a chair. Obviously, there will be one person who doesn't get a chair. That person is now out of the game, one chair is removed from the row, and the music and walking begin again. This continues, with one person being eliminated and one chair removed each round, until there is only one person left, who is declared the winner.

   (a) If there are 5 children at the party, how many ways can they be seated in the chairs when the music stops for the first time?

   (b) If there are $n$ children at the party, how many ways can they be seated in the chairs when the music stops for the first time?

(c) If there are 6 children at the party, one of whom is named Max, in how many different orders can they be eliminated during the game if you know that Max will be eliminated first? (All the children, including the winner, must be in the order.)

(d) If there are $n$ children at the party, one of whom is named Mindy, in how many different orders can they be eliminated during the game if you know that Mindy will be the winner? (All the children, including the winner, must be in the order.)

(e) Suppose there are four children playing: John, Kate, Lisa, and Mike. You know that John is a better at this game than Mike, and Lisa is better than Kate. Suppose you also know that in playing musical chairs, if player $X$ is better than player $Y$, then $Y$ will be eliminated before $X$. Using this knowledge, draw a possibility tree showing the possible orders in which the children can be eliminated (so the winners will be at the leaves of the tree).

How many different orders are there?

4. Suppose a group of six students attend a concert together.

(a) How many different ways can they be seated in a row?

(b) Suppose one of the six has to leave the concert early to finish a CS172 homework assignment. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

(c) Suppose the six students consist of three boyfriend-girlfriend couples and each couple wants to sit together so that the boy is on the right. How many ways can the six be seated?

(d) Suppose the six students consist of three math majors and three CS majors. Each group of majors wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

5. Simple combination locks are opened by dialing a certain sequence of three numbers on a dial. Assume that the same number may appear twice in a combination, but not sequentially. That is, the combination 13-20-13 is permissible, but not 20-13-13. Assuming every number in a combination must be between 0 and 50 (including 0 and 50), how many possible combinations are there?

6. Rhodes College surveyed 100 prospective employers as to which programming languages they wanted their new hires to know.

   28 employers said they wanted new hires to know Python.

   26 employers said they wanted new hires to know C++.

   14 employers said they wanted new hires to know Java.

   8 employers said they wanted new hires to know Python and C++.

   4 employers said they wanted new hires to know Python and Java.

   3 employers said they wanted new hires to know C++ and Java.

   2 employers said they wanted new hires to know all three languages.

(a) Draw a labeled Venn Diagram corresponding to this situation (label all 8 regions).

(b) How many employers wanted students to know at least one of the languages?

(c) How many employers wanted students to know Java and Python, but not C++?

(d) How many employers wanted students to know Python and C++, but not Java?

7. Rhodes is going to send a group of computer science majors to a local high school to talk to the high schoolers about how cool CS is.

   (a) There are 20 CS majors. How many ways can a group of 5 be picked to visit the school?

   (b) The 20 CS majors consist of 12 first/second-year students and 8 third/fourth-year students. The group of 5 to visit the school should consist of at least one first/second-year student and at least one third/fourth-year student. How many ways can the group be picked?

   Hint: Use the difference rule or the addition rule.

   (c) A group of 5 is picked at random (not following the guidelines from part (b)). What is the probability it consists of all first/second-years or all third/fourth years?

   (d) Two other high schools get on board and want a group of 5 CS majors to visit. So now you need to pick 3 groups of 5 students each to send to the three schools. Note that it matters which group goes to which school, but within each group, the ordering of the students doesn't matter.

   Hint: Call the schools A, B, and C. First, pick the students to visit school A. Then pick the students to visit school B. Then pick the students to visit school C.

8. In the poker variant of Texas Hold'em, there are multiple betting rounds. In the first round, each player is dealt two playing cards at random from a standard deck of 52 cards. For these problems, the order of the cards doesn't matter.

   (a) If you are dealt two cards at random from a standard deck of 52 cards, how many possible ways can this be done? That is, how many possible two-card hands are there?

   (b) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards are both face cards? (Face cards are jacks, queens, and kings. The two cards don't have to be the same face card.)

   (c) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability you have a *pocket pair*, meaning the two cards are of the same rank? (e.g., both queens, both fives, both aces, etc)

   (d) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your hand is *suited*, meaning the two cards are of the same suit? (e.g., both diamonds, two clubs, etc)

   (e) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards match in rank and suit?

   (f) If you are dealt two cards at random from a standard deck of 52 cards, what is the probability your two cards have different ranks and different suits?

9. Suppose I pick an arbitrary integer $n$, which doesn't change once I pick it for this problem. Let $S$ be the set $\{0, 1, 2, 3, \ldots, 2n-1, 2n\}$ (in other words, $S$ is the set of integers between 0 and $2n$).

   (a) If I choose $n+1$ integers from $S$, must at least one of them be odd? Why or why not?

   (b) If I choose $n+1$ integers from $S$, must at least one of them be even? Why or why not?

10. Suppose I pick three integers arbitrarily. Use the pigeonhole principle to explain why among those three integers, there must be a pair of integers whose difference is even. (You may state whatever facts you want about even or odd numbers without proof, as long as your statements are true.)