Discrete Structures, Spring 2016, Homework 7

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

Prove the statements below. They are all true. Refer to sections 4.4 and 4.5.

1. The product of any two consecutive integers is even.

   Hint: Use the quotient-remainder theorem with $d = 2$.

2. The product of any two consecutive integers can be written as $3k$ or $3k + 2$ for some integer k.

   Hint: Use the quotient-remainder theorem, but see if you can determine on your own what d should be.

3. For any integer n, $n^2 + 5$ is not divisible by 4.

   Hint: Do this by contradiction. Use the Q-R theorem, but you don't need to use $d = 4$; a smaller d works. Each case in the Q-R theorem should lead to a contradiction.

Do the following problems about sequences and series. Refer to section 5.1.

4. Write out the first four terms for each of the following sequences. List the name of the variable, the subscript, and the number itself. For example, for "$\forall n \in \mathbb{Z}^{\geq 1}\ d_n = 2n$" you would write "$d_1 = 2,\ d_2 = 4,\ d_3 = 6,\ d_4 = 8$."

   (a) $\forall i \in \mathbb{Z}^{\geq 2}\ a_i = i(i-1)$

   (b) $\forall j \in \mathbb{Z}^{\geq 0}\ s_j = \dfrac{j}{j!}$

   (c) $\forall k \in \mathbb{Z}^{+}\ z_k = (1-k)(k-1)$

5. Reduce each of the following expressions to a single numeric value.

   (a) $\displaystyle\sum_{j=1}^{5} \frac{(-1)^j}{j}$

   (b) $\displaystyle\prod_{k=0}^{10} \frac{10-k}{2^k}$

   (c) $\displaystyle\prod_{i=1}^{3} \left(\sum_{j=i}^{3} i \cdot j \right)$

6. Write the following sums using sigma (Σ) notation.

   (a) $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \cdots + \dfrac{n}{(n+1)!}$

   (b) $\dfrac{n}{1} + \dfrac{n-1}{2} + \dfrac{n-2}{3} + \cdots + \dfrac{1}{n}$

   (c) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$